Generation and Exploration of Architectural Form Using a Composite Cellular Automata

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Abstract. In this paper, we introduce a composite Cellular Automata (CA) to explore digital morphogenesis in architecture. Consisting of multiple interleaved one dimensional CA, our model evolves the boundaries of spatial units in cross sectional diagrams. We investigate the efficacy of this approach by systematically varying initial conditions and transition rules. Simulation experiments show that the composite CA can generate aggregate spatial units to match the characteristics of specific spatial configurations, using a well-known architectural landmark as a benchmark. Significantly, spatial patterns emerge as a consequence of the evolution of the system, rather than from prescriptive design decisions.

1 Introduction

The production of high density housing in many large cities has typically focused on optimizing the use of space, disregarding the quality of the inhabitable spaces being built. Attributes such as access to sunlight, ventilation, and storage space, which are generally regarded as essential for 'better living' [23], have often been overlooked. In response to the increased development of living spaces that are commonly perceived to be sub-standard [11], new urban design rules and regulations have recently been proposed in Melbourne, Australia. From a design perspective, the introduction of revised planning rules provides the impetus to investigate new methods for the creative exploration of design space in search of novel ways to produce liveable spaces.

In this paper, we introduce a 'digital morphogenesis' method to tackle this design challenge. Here, a composite cellular automata (CA) consisting of multiple, regularly spaced interleaved 1D CA provides the structure for a designer to interactively 'generate and explore' the design search space. The composite CA includes a combination of 'self-assembly,' 'pattern formation' and 'best variant' selection to produce, in this case, cross sectional diagrams of spatial configurations. Metrics for the evaluation of emergent attributes of the spatial configurations are introduced in order to allow the designer to interactively select instances that satisfy the requirements of the task in unexpected ways, potentially leading towards a novel manner of representing and understanding the design.

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Our approach represents a departure from the oversimplification that the 'form–follows–function' paradigm, strongly enforced on the design practice during the modern movement [6]. The rationale behind our 'bottom-up' design methodology is to define a way in which low-level design elements [20] interact in, and with space, in order to enable the exploration of design solution space, rather than focusing on optimizing a solution based on a fixed set of requirements. Detailed simulation experiments demonstrate a proof-of-concept that our composite CA model can automatically synthesize shape and topology, *in silico*, producing abstract diagrams of spatial configurations that, given the characteristics of the constituent elements (building blocks), can be easily translated into architectural cross sections.

The remainder of this paper is organised as follows. In Sect. 2, we introduce work related to computational morphogenesis and generative design. This is followed by a formal description of CA and a brief review of CA in architectural design. Our model is introduced in Sect. 3. In Sect. 4, the simulation experiments are described and results presented. We summarise the results and discuss the implications of our findings, before briefly outlining avenues for future work in Sect. 5.

2 Background

2.1 Computational Morphogenesis

Generative systems have been used to investigate novelty in architecture and urban design since Aristotle [22, p. 30]. Beyond classic examples of generative systems (Greek orders, Da Vinci's central plan churches, Durand's elements, etc.) there are examples of form generation techniques often used in architecture and urban design in the twentieth century, e.g. Alexander's work with 'patterns' [1] and Stiny's 'shape grammars' [28].

Computational (or digital) morphogenesis techniques, use digital media as a generative tool for the derivation of and manipulation of 'form' [12,13], where abstract computer simulations are used to foster the gradual development and adaptation of shapes [29]. Using bottom-up generative methods, they combine a number of concepts including self organization, pattern formation, self-assembly and 'form-finding.' Self-organization is a process that increases the order and statistical complexity of a system as a result of local interactions between lower-level, simple components [4,26]. Emergence represents the concept of the patterns, often unpredictable ones, which form in large scale systems [16,21]. Emergent properties arise when a complex system reaches a combined threshold of diversity, organization and connectivity. For example, the self-assembly of geometric primary elements (or 'building blocks') may, in some systems, be an emergent form-finding property guided by strict rules dictating 'bonding' patterns [8,17].

2.2 Cellular Automata

CA are discrete dynamical systems comprising a number of typically identical simple components (or cells), with local connectivity over a regular lattice whose global configuration changes over time, according to a local state transition rule. CA implementations and functions, regardless of their complexity, regularity and constraints, require the definition of characteristics (cells, cell-states and neighbourhood) that can be directly interpreted as spatial configurations. Formally, a CA is defined by:

- an array of cells of length L^D (where D is the number of dimensions)
- a neighbourhood size n for each cell $c \in L$
- an alphabet of cell states $\Sigma = \{s_i, \ldots, s_{|\Sigma|}\}$
- a discrete time step $t = 0, 1, \ldots$
- a state $s(c, t) \in \Sigma$ for each cell $c \in L$ at time t
- a transition function $\psi : \Sigma^{|n|} \rightarrow \Sigma$

At time $t + 1$, the state of each cell c is updated in parallel using the transition function and the defined local neighbourhood n. For an elementary 2 state 1D CA with $n = 3$ neighbours, there are $2^8 = 256$ possible transition rules. For a 2 state 2D CA with $n = 4$ neighbour (von Neumann neighbouhood) there are $2^{32} = 4 \times 10^9$ possible transition rules. The number of rules can be reduced if different symmetries are adopted. However, as the number of states and neighbourhood size increase, the state space significantly increases.

CA can be seen as a space for exploratory creativity. Von Neumann [30] showed that CA may produce very sophisticated self-organized structures, given a finite number of cells states and short range interactions.

CA have been used effectively to help explain natural phenomena involving strong and explicit spatial constraints [32,33]. They have been used to model morphogenesis processes [25], and as a model to generate simple shapes [7], or specific 2D or 3D target patterns [5]. CA have also been used as part of a more general 'meta-design' design process in engineering [9,18].

2.3 Cellular Automata and Design

In architecture, 3D implementations of CA have been typically used to produce diagrams of abstract spatial configurations that can serve as starting points for the further development of architectural or urban form. The cells of the CA represent 3D spatial units with programmatic characteristics (e.g., housing units, rooms, public spaces, circulation spaces, etc.), which results in functionally deterministic outputs.

Coates *et al.* [6] present a 3D model using cubic cells with binary states ('occupied'/'empty') in search for emergent patterns, emulating the work of Conway and his 'Game of Life' [10]. For this purpose, he explores a series of rule combinations and neighbourhoods. The aim of these experiments was to find mechanisms for the generation of spatial structures with potential to be used in architectural design. Krawczyk [19] uses a similar implementation of 3D CA to evolve spatial configurations, focusing on how can the abstract outputs of the model be translated into architectural form. The translation is performed by manipulating the characteristics of the cells once the model has stopped running, which brings this approach closer to a more traditional design process. Here, the CA time evolution is presented as an exploration, where desired outcomes or other parameters that allow for the evaluation of the system's performance are not defined.

Herr and Kvan [14] present a different approach, where the constraint of a fixed, regular lattice for the CA is removed and the designer may interact with the time evolution of the system, steering the evolution of the CA according to design goals. This approach integrates the shaping of a design solution with the reformulation of the design problem, thus reducing the post-processing of outcomes to detailing. Araghi *et al.* [2] describe the use of CA in the development of high density housing where the generation of variety based on additional design objectives (accessibility and lighting) is the goal. The design requirements are mapped to cell states within the local neighbourhood, and the transition rules inform the development of the system. The definition of 3D cells implies a design operation that binds the form of the cell to a particular function, which renders the results of the development of said models functionally static.

3 Model

Our composite CA is a digital morphogenesis tool that can be used at the early stages of an architectural design process. The composite CA is built as an array of evenly spaced interleaved 1D CA (Fig. 1a), arranged on a grid (Fig. 1b). With this arrangement it is possible to produce spatial configurations where the 'cells' of the CA have a 'form-making' role, rather than being functionally predefined. Our approach focuses on how space can be physically reshaped and characterised as the system evolves, which represents a departure from the typical use of CA

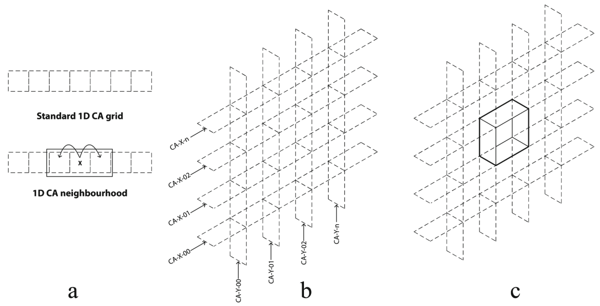

Fig. 1. (a) A standard 1D CA. (b) The configuration for our composite 1D CA consisting of interleaved horizontal and vertical 1D CA. (c) A representative example of one spatial unit, defined by the activation of its boundaries.

in architecture and urban design, where the characteristics of the space are prescribed by design.

What differentiates our composite CA from a standard 2D CA is the fact that the multiple 1D CA act as the edges of encapsulated 'spatial units' (Fig. 1c). That is, each edge of a spatial unit is actually a discrete cell in a 1D CA and is governed by a state transition rule. Here, each cell has a binary state – it can be either active (on) of inactive (off). If a cell in a 1D CA is off, the spatial units on either side of it are connected. System dynamics generate 'complex' patterns consisting of concatenated spatial units, defined by active/inactive edges. The emergent structures are highly sensitive to individual cell states and transition rules, a system with some similarities to bond percolation models and abstract genetic regulator systems [31].

In our composite CA, there are two possible states for each cell. Given the configuration of the interleaved 1D CA, this results in 16 different possible configurations for each of the encapsulated spatial units, illustrated in Fig. 2.

In Fig. 3, we show representative examples of the complex spatial topologies that emerge as a result of the concatenation or combination of multiple edges being active/inactive at the same time, which illustrates the exploratory power of the model. In Fig. 3b, we label the centre of each individual spatial unit and

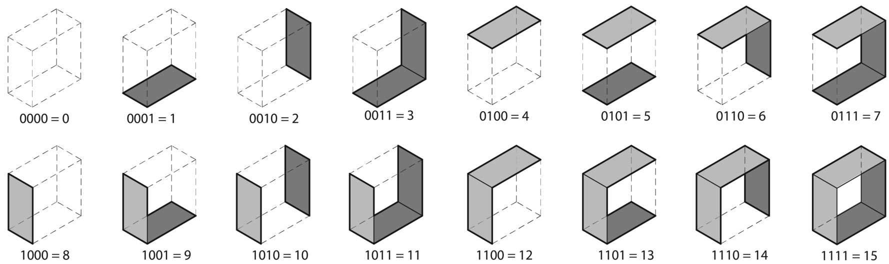

Fig. 2. 3D representation of the 16 spatial configurations the model is capable of producing for a single 2D spatial unit. Binary counting is used to number the active edges.

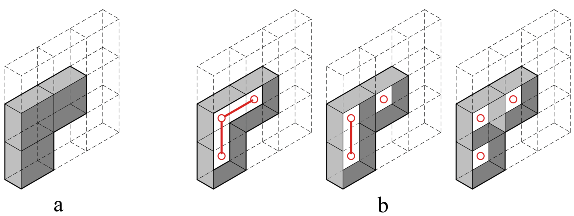

Fig. 3. (a) A standard 2D CA, where each cell is a spatial unit in itself (3 cell configuration). (b) 3D representation of three possible spatial unit configurations of size 3 units that can be produced with the proposed composite CA model. The centre of each spatial unit is labelled with a red circle (node). Connecting spatial units are also shown (edges). (Color figure online)

include connecting edges between adjacent spatial units where appropriate. It is this formation of aggregates or clusters of connected 2D ‘encapsulated spatial units’ that subsequently generates a volumetric matrix for spatial organisation to be used by the designer.

Unlike a traditional 2D CA, where the characteristics of the cells are defined by their state, in the composite 1D CA, spatial units are neutral, and acquire their characteristics depending on the configuration of their boundaries.

4 Experiments

A series of simulation experiments were carried out to evaluate the efficacy of the proposed composite CA model, focussing specifically on the configuration and characterization of space. The key question guiding the experimental design: *Can the composite CA be used to effectively generate diagrammatic cross-sections of architectural form?*

4.1 Methodology

We start by systematically examining the dynamics of instantiated instances of the composite CA by varying the initial conditions of each CA and transition rules. We then examine whether the composite CA can generate (evolve) aggregate spatial units, with specific spatial attributes, corresponding to configurations representing a mix of open and closed spaces.

Parameters. The composite CA consists of $x \times y$ regularly spaced 1D CA, where x and y correspond to the number of cells (L) in the corresponding horizontal and vertical 1D CA. We examine $L = 10$. We set the local neighbourhood size $n = 3$, and limited the alphabet of cell states to $\Sigma = \{0, 1\}$ (i.e. the cell representing the boundaries of the spatial units are either active or inactive).

The state transition rules are drawn from Wolfram’s [32] elementary 1D CA rules – representative rules from classes III and IV are used, where Class III (random) contains rules that generate outcomes with no discernible patterns and Class IV (complexity) contains rules that generate discernible patterns that repeat at unpredictable frequencies and locations, as the system develops. Classes I (uniformity) and II (repetition) have been disregarded at this stage, since they tend to yield configurations that become static in time.

We use a different state transition rule for each of horizontal and vertical 1D CA. From class III we selected rules 30 and 60. From class IV we selected rules 54 and 110 (other rules were tested but are not reported in this paper).

In order to allow the experiments to generate a variety of spatial configurations, each simulation trial was run for a maximum of 200 time steps, starting from uniformly randomly drawn initial cell states. The entire system is updated simultaneously in discrete time steps.

Analysis. We introduce a phenotypic diversity measure on the space of the composite CA to analyse emergent behaviour. Specifically, we examine the embedded 'connectivity graph' where nodes within the graph correspond to the centre of active adjacent spatial units in the model (see Fig. 3b). The structure of connected nodes define a 'local cluster' or clusters of adjacent spatial units, possibly corresponding to arbitrarily shaped geometric forms, defined by active/inactive cells of the composite CA. This graph-based analysis provides a concise way to examine the spontaneous formation of 'motifs' that represent a wide variety of spatial attributes. Clusters act as a conduit for circulation through different interconnecting spatial units and provide a balance between the open and closed space. It is worth noting that some of the nodes are located outside the boundaries of the $x \times y$ 'lattice'. When a cluster has one of its nodes with that condition, it is considered an open cluster.

We use three graph theoretic metrics to characterize the emergent dynamics for specific rules and time-evolution of the composite CA: **M1** the degree distribution of nodes – the regularity of the aggregation of spatial units (where a low degree distribution represents a more irregular spatial configuration); **M2** the mean and standard deviation of cluster size – quantifies the level of fragmentation of space; and **M3** the ratio of the number of open and closed clusters (where a cluster is considered open when it has one or more nodes outside of the lattice) – quantifies porosity or the connectivity of the spatial configurations to the exterior.

4.2 Results

Time Evolution of the Composite CA. Snap-shots of the evolving connectivity graphs, corresponding to the emergent spatial forms for two different rule combinations at time steps $t = (50, 100, 150, 200)$, are shown in Fig. 4. It is inter-

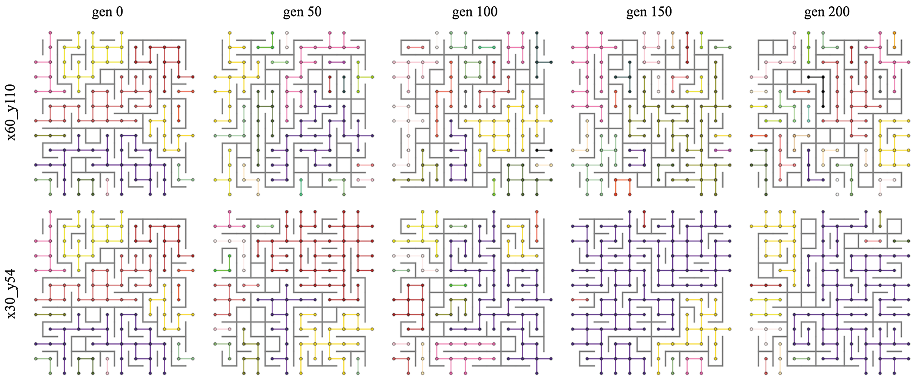

Fig. 4. Snap shots of the evolving composite CA. The top and bottom rows show the connectivity graphs at times t = 0, t = 50, t = 100, t = 150 and t = 200 for rule x60_y110 and x30_y54 respectively. Note that some of the nodes are outside of the lattice (Color figure online)

esting to note the variety of cluster sizes and shapes that are being generated, which provides a wide search space for exploring spatial attributes.

The emergent spatial unit structure – represented by clusters – change shape significantly over the course of the simulated evolutionary time, to a point where there is no apparent relationship between generations evolved using a particular set of rules. For instance, looking at rule combination $x60\_y110$ (Fig. 4, top row), after 50 generations it is possible to observe an aggregation of similarly sized shapeless clusters, where the most recognisable elements are the $size = 2$ closed clusters. However, looking at generation 100 of the same rule combination, it is possible to note the re-appearance of closed $size = 4$ formations, also found at time step $t = 0$, which exist either as closed clusters or as part of larger ones. These formations can be interpreted as large, regular empty spaces, which differentiates them from other formations by their attributes – they can be thought of as motifs. Similarly, looking at time step $t = 50$, in the snapshots corresponding to rule combination $x30\_y54$ (Fig. 4, bottom row), close to the top right corner, it is possible to observe a series of formations cycling around a single boundary, which could be interpreted as a large subdivided regular area, providing a different set of spatial attributes. It is important to note that all these new instances are generated by the same structural constraints, or transition rules.

To conclude the preliminary analysis, we plot time series values of the cosine similarity metric (Eq. 1) between the evolving spatial configurations at each time step of the simulation in Fig. 5.

$$\text{similarity} = cos(\theta) = \frac{\sum_{i=1}^{m} \mathbf{V}_{i,(t)} \times \mathbf{V}_{i,(t+1)}}{\sqrt{\sum_{i=1}^{m} \mathbf{V}^2_{i,(t)}} \times \sqrt{\sum_{i=1}^{m} \mathbf{V}^2_{i,(t+1)}}} \tag{1}$$

Here, $\mathbf{V}$ is a vector of graph theoretic metrics of length m, $\{\mathbf{M1,\ M2,\ M3}\}$. The vector evaluated at consecutive time steps. An inspection of the plot provides additional supporting evidence for the gradual transition between alternative spatial configurations. However, what is most interesting is the sudden spikes/drops in similarity values (e.g., at $t = 100$ for $x30\_y60$) over the course of

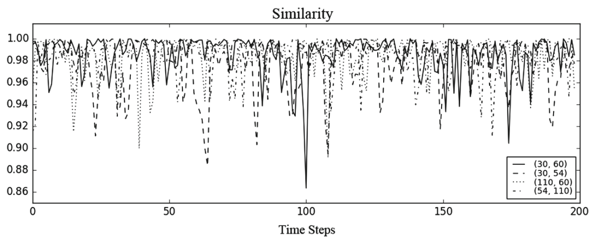

Fig. 5. Cosine similarity *vs* time, where the vector of feature at each time corresponds to average cluster size, std. dev for average cluster size, open clusters/closed clusters ratio.

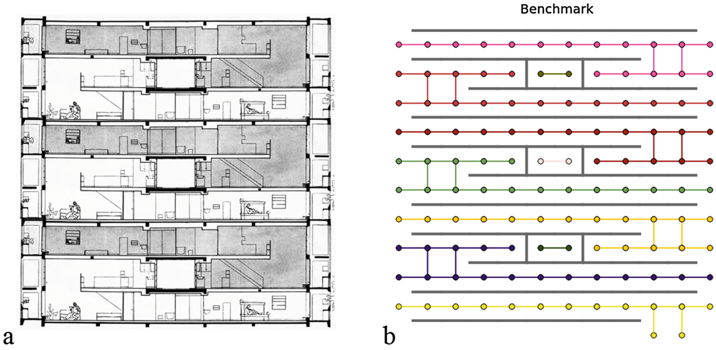

Fig. 6. Typical section of 'Unitéd'habitation' by Le Corbusier (a) and its representation as connectivity graph (b), generated using the alphabet of 16 possible spatial units illustrated in Fig. 2. (Color figure online)

the time evolution of the model – reminiscent of 'punctuated equilibria,' consistent with innovative/adaptive behaviour [24].

Attribute Matching. In the second phase of our analysis, the goal was not to match any given spatial pattern exactly, but rather to investigate whether 'interesting' smaller building blocks (correspond to local cluster or motifs) could be evolved. The emergent abstract spatial configurations would then be translated into architectural cross sections as part of the early stage of design. As a benchmark, the typical section of the interlocking dwelling units of the 'Unitéd'habitation' by Le Corbusier is used (see Fig. 6). This choice of benchmark was motivated by its formal characteristics that allow for a series of potentially desirable attributes in terms of lighting, ventilation and circulation performance

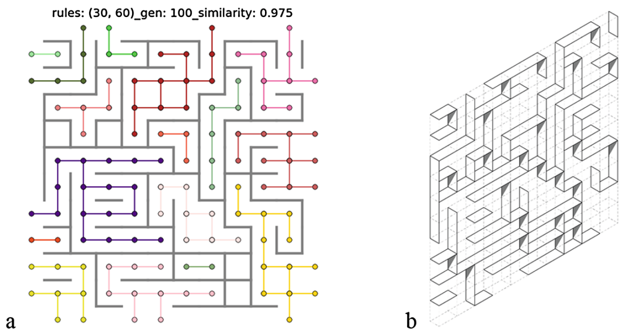

Fig. 7. (a) Connectivity graph for evolved spatial configuration with cosine similarity value = 0.975 corresponding to the typical section of Fig. 6. (b) 3D representation of the evolved connectivity graph, which brings the abstract output of the model to a language that can be easily interpreted from an architectural perspective. (Color figure online)

that could be further investigated as input parameters to be implemented into the proposed system.

The plot shown in Fig. 7(a) illustrates an example of emergent spatial form, with a high similarity value, generated by our composite CA. A cosine similarity value of 0.975 was found using Eq. 1 where A was the benchmark connectivity graph shown in Fig. 6(b) and B was the evolved connectivity graph in the plot. Significantly, Fig. 7 illustrates a variety of 'forms', which can be detailed, developed or interpreted by a designer at a later stage, where implicit meanings of the overall structure and boundary elements of an architectural space are expanded upon. Figure 7(a) depicts a 3D representation of the plot in Fig. 7(b), which brings the abstract output generated by evolving the model, into a language that can easily be interpreted and recognised by architectural designers as a spatial configuration to be further developed and detailed.

5 Discussion and Conclusion

In this paper, we have described a composite CA that can be used to generate a variety of spatial configurations by defining the boundaries of 'encapsulated spatial units,' as well as their interconnections. The characteristics of the generated space emerge as a consequence of the evolution of the CA, rather than being prescribed by design, as properties of the cells, as it happens with more common implementations of CA in architecture and design. Our goal was to explore the formation of aggregates or clusters of encapsulated spatial units, in search for 'interesting' spatial organizations with potential to be detailed, developed and/or interpreted by a designer at a later stage. Our model was able to produce clusters of a wide variety of sizes, shapes and with different 'spatial attributes' (regularity, openness, fragmentation, among others). We have described metrics that can be used to evaluate the emergent patterns against design criteria, which for the moment can only take the form of aggregations of fixed configurations (see Fig. 2). Our digital morphogenesis approach seeks to maintain both flexibility and fluidity, as it is required for creative design exploration.

It can be argued that the strength of the composite CA system is based on its capability to produce a vast array of configurations that can be evaluated in terms of their characteristics. In this paper we have shown the analysis of a few rule combinations, selected from different classes, in order to demonstrate the efficacy of the approach. However, it appears reasonable to expect different results if different rules are used.

With all this being said, our composite CA system can be described as a tool that provides designers with a range of alternatives to satisfy given design requirements, rather than acting as a direct design tool for completed design solutions. In its current state, the ability of the model to generate/search the state space is defined by transition rules and the time evolution of the model. In our experiments, the benchmark target was a pre-defined spatial configuration. However, we found that searching for a fixed, static configuration limited the possibilities by constraining the desired output to what has already been imagined by other designer, defeating the ultimate purpose of the model – generating a design space, and searching through it using design criteria, looking for emergent spatial configurations. Therefore, introducing protocols to search for characteristics of the space (e.g., open *vs.* closed space, or mean cluster size), rather than specific fixed patterns, is seen as a strategy that suits the purpose of enabling the emergence of unexpected spatial configurations. In this regard, the development of more accurate metrics to represent 'spatial attributes', the development of mechanisms to incorporate modifications to the rules as the system evolves, as well as the introduction of external influences, are seen as plausible paths to pursue in order to extend the system's capabilities.

The graph theoretic analysis of the composite CA time evolution has some similarities with concepts from 'space syntax' [15,27]. In space syntax, graphs are used to represent the sub-divided space in order to identify specific configurations, which are then analyzed via social relations and properties. In contrast, in our approach we search for configured space in terms of physical attributes, which may be understood as a connected set of discrete units, rather than a continuum [3]. This configure space then acts as input into subsequent evolutionary cycles in a search for new, emergent, spatial configurations.

There are many opportunities to extend this work. One interesting direction would be to 'fine tune' the metrics to better reflect design requirements. Another avenue is to explore the use of evolutionary algorithms to search for design 'motifs' encapsulated by specific metrics and to examine design trade-offs.

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